# PARALLEL WORLDS <br> A response to <br> Timothy Easton's article in SPAB Magazine, Winter 2016 



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This article was written in response to Timothy Easton's article Parallel Worlds published in SPAB Magazine, Winter 2016 This eDITION 2017

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## Parallel Worlds: an alternative view

I was particularly interested to see so many examples of divider scribed geometrical symbols because I have recorded, analysed and published articles on them myself for over 30 years and have just been asked to speak on this topic at the Carpenters' Fellowship Frame Conference at Saint Fagans National History Museum in August this year.

Whilst Timothy Easton's article presents the case for the apotropaic use of a wide range of symbols it also raises two important questions in relation to the divider scribed geometrical symbols,

1 Who scribed the geometrical symbols?
2 What other functions might geometrical symbols have?
Timothy Easton answers the first question by recognising that the geometrical symbols were scribed by frame carpenters and masons, the craftsmen who actually used dividers in their daily work, in the design and construction of buildings. However, the practical function of dividers and the symbols and geometries that can be derived from them was not explored.

The second question is answered by my own research into the construction and application of proportional geometries in building design. Analysing the same geometrical symbols, and the proportional symmetries that they embody, reveals them as elements of a spatial design language used in the design of buildings and other artefacts throughout the medieval period. The following drawings and text give evidence of geometrical building design that evolves from two of the symbols in the article (images 4A and 4B on page 44 of the SPAB Magazine, Winter 2016) which are redrawn below.

(Figure 4A)


2 FIVE CIRCLE GEOMETRY (Figure 4B)

Symbol 1, which is described in the article as a hexfoil, is more commonly known by modern frame carpenters as a daisy wheel. The primary circle (the full outer circle) and it's six secondary internal arcs are all drawn to the same radius. Symbol 2 , which is described in the article as a consecration cross, is a five circle geometry. Like the daisy wheel, it'sprimary circle and the four internal arcs are all drawn to the same radius. Both symbols can be drawn at any scale, at small scale by conventional dividers or at large scale by trammel. Whatever the scale, the geometry retains its proportional symmetry. Despite being compass drawn or divider scribed and therefore circular, both symbols are the source of angular geometrical plane constructions, drawing 1 generating a variety of triangles and two rectangles and drawing 2 generating the perfect square and octagon. The daisy wheel is also a source of precision $30^{\circ}, 60^{\circ}$ and $90^{\circ}$ angles and further angles constructed by halving or addition. These constructions are shown in the following drawings.


3 EQUILATERAL TRIANGLE


4 ALTERNATE POINT TRIANGLE


5 THE 4 POINT RECTANGLE


6 THE 6 POINT RECTANGLE


7 HEXAGON

## Geometrical properties of the Daisy Wheel

Drawings $3,4,5,6$ and 7 show a range of precision plane areas that can be derived from the daisy wheel by drawing lines along a straight edge between the wheel's axis (in figure 3) and petal tips (in figures 4, 5, 6 and 7). It can be seen in figure 3 that the equilateral triangle is one of six that radiate around the wheel's axis and that all six combined form the hexagon in drawing 7. The large equilateral in drawing 4 is one of two that can be drawn within the wheel, each one pointing in the opposite direction. The two overlapping equilaterals combine to form the 6 point star of David. The linear boundaries of the equilaterals intersect to form a small hexagon at the centre of the star. Drawings 5 and 6 show the two rectangles that can be constructed within the daisy wheel. Both can be drawn either horizontally or vertically, depending on the axis of the daisy wheel. It can be seen that when the wheel has a vertical axis (as in the drawings) it generates a horizontal 4 point rectangle and a vertical 6 point rectangle. Both of these rectangles can be

1.73205080757


8 THE 4 POINT RECTANGLE
found in the floor plans and sections of medieval buildings as will be shown further on. Drawing 8 shows the 4 point rectangle, known in modern parlance as a root 3 rectangle (because, with a short side of 1 it has a long side of 1.73205080757 ad infinitum, the indefineable square root of 3 ) has some other interesting properties. If it is halved along its diagonal it generates a perfect set square with the ratio 1:2 between it's short side and diagonal. The ratio is logical because the rectangle's short side distance between the daisy wheel's petal tips on the primary circle's circumference is equal to the circle's radius and the rectangle's diagonal is equal to the diameter. The angles of the set square are $30^{\circ}, 60^{\circ}$ and $90^{\circ}$, a perfect frame carpenter's square.


Drawing 9 shows how tangents to the head and foot of the daisy wheel and parallel alignments through it's axis and petal tips, divide the wheel into the four identical horizontal bands shown on the left while, in the context of church design, the bands can be configured as the three bands shown on the right where the two inner bands are combined in the greater width of a nave, as is the case in the nave of Ely Cathedral, described further on. Drawing 10 shows the daisy wheel's function as a protractor. The dashed arcs subdividing the angles can be drawn as part of the wheel or drawn later. The angles can be added so that $30+15=45$ and $60+90=150$, etc.



11 FIVE CIRCLE SYMBOL (Figure 4B)


12 FIVE CIRCLE SQUARE GEOMETRY


13 FIVE CIRCLE OCTAGON GEOMETRY

## Geometrical properties of the Five Circles

The symbol shown in drawing 11 (4B in Timothy Easton's article where it is described as a consecration cross) is a shorthand version of five circle geometry which, in turn, is a preliminary stage in the construction of the five circle square. Drawing 12 shows the construction of the five circle square, comprised of a primary circle and four polar circles of identical radius. (the upper and lower circles are drawn as shortcuts but intersect at the corners of the square. The left and right circles are shown complete). To draw the polar circles in the correct locations the geometry must be set out on perpendiculars with the primary circle's axis located at their intersection. The polar circles are then drawn from where the perpendiculars cut the primary circle. Linking the four points of intersection outside the primary circle generates the perfect square. This is the fastest and most accurate way to draw a square manually. Squares can be drawn easily to controlled sizes because the side of the square is equal to the diameter of the primary circle, so a ten inch square requires a ten inch diameter circle, drawn with the dividers set to a 5 inch radius.

Drawing 13 shows how a large circle is drawn from the primary circle's axis to pass through the corners of the square. Where the perpendiculars cut this larger circle a second square (or diamond) is drawn at $45^{\circ}$. The square and diamond overlap to form an octagon and an eight point octagon star.


## Geometrical properties of the Vesica Piscis

The symbol shown in drawing 14 (4C in Timothy Easton's article where it is titled eclipse) is a shorthand version of the vesica piscis. The dictionary definition of vesica piscis is fish bladder, from it's fish like shape when horizontal and it's similarity to the section of a fish when vertical. Drawing 15 shows how the vesica piscis is drawn as two circles of identical radius are drawn on a horizontal line with the circumference of each passing through the axis of its neighbour. A vertical line drawn through the intersections of the circles is automatically a perpendicular generating $90^{\circ}$ right angle at the intersection. This construction is a simple, accurate and rapid carpenter's method for marking out right angles on timbers about to be cut. Because the two circles are drawn to pass through each other's axis and because their radii are identical the construction divides the horizontal line within the circles into three equal sectors. This doesn't look correct because the vesica curves inwards towards its points and seems to get narrower while the circles at either side swell out into the large moon shapes that give the symbol its name. However, although the visual image is true in two dimensions and the two moons do actually have larger areas than the vesica, the linear divisions along the horizontal line are equal.

## The Daisy Wheel design of anglo-Saxon water mill

Work on the construction of the high speed Channel Tunnel Rail Link between England and France, carried out between 1988 and the project's completion in 1994, generated many opportunities for archaeological exploration along the route from London through Kent to the English terminal at Folkestone. The site of the new Ebbsfleet station, between Dartford to the west and Gravesend to the east, south of the Thames and north of the A2 (Roman Watling Street) yielded a number of exciting discoveries. A Straight Tusked Elephant skeleton over 400,000 years old was found in the sediments of a small lake and an opulent Roman Villa and Anglo-Saxon water mill were discovered at Northfleet, just downstream from Ebbsfleet. The water mill is the most exciting of these discoveries for its remains have been dendro-chronologically dated to the late seventh century (between 684 and 720 AD) which, according to Wessex Archaeology, is the earliest recorded horizontal wheel tidal water mill in Britain. Even more exciting, from a geometrical perspective, was the discovery of a delicately scribed compass geometry on one of the mill's pentroughs: long, tapered boxes that direct the force of water onto the mill's radial paddles to generate rotation.

While this geometrical symbol may have been scribed as a ritual protection it can also be recognised as a design icon that yields the form of the wheel, as the following drawings show.

Drawing 16 shows a scan of a tracing that was made available to me by archaeologist Damian Goodburn, a specialist in historic water environment timber constructions. The compass geometry was finely scribed by dividers into a chute board at Ebbsfleet Mill and is an extraordinary survival because it reveals a hitherto missing link in the evolution of compass geometry in England. Although fragile and incomplete, the construction is clear enough to be reconstructed with absolute certainty. The geometry's first stage is simple, a primary (central) circle with six further circles drawn to the same radius around its circumference, drawing 17.


16 SCAN OF TRACING


17 THE FULL DAISY WHEEL

Damian explained that the mill chute timber had survived for almost one thousand three hundred years because, being submerged in mud, it was protected from the air that would have caused it to decompose. When discovered, the timber was saturated and its surface coated with mud, the inscribed geometry was difficult to see (and therefore difficult to record photographically) and so fragile that the timber's surface could not be cleaned to reveal it. The solution was to trace the image avoiding any direct contact. Damian also explained that while the mill chute timbers had survived, the axis or hub of the mill had not and the number of radial paddle arms was therefore unknown. However, in another horizontal water mill in the north of England, archaeologists had found the reverse with the hub surviving but the chutes missing, the hub (similar to a cartwheel hub) having twelve paddle arms radiating from it. I was able to show Damian that daisy wheel geometry enabled the design of mill wheel hubs with either six or twelve paddle arms.

Drawing 18 shows that the arcs of the six peripheral circles intersect each other at twelve points, six outside the central, primary circle and six exactly on the primary circle's circumference, within which they generate the familiar daisy petal pattern. Lines drawn through the inner and outer intersections divide the $360^{\circ}$ circumference of the wheel into twelve equal $30^{\circ}$ sectors and are simultaneously the radials that define the locations of the paddle arms. Connecting three points of intersection on the primary circle's circumference generates a $30^{\circ} 60^{\circ} 90^{\circ}$ right angled triangle (in blue tone), a perfect carpenter's set square with the harmonic ratio 1:2 between its short side and diagonal. So the Ebbsfleet daisy wheel geometry can be used to construct a protractor and a set square, the two essential tools for precision carpentry layout of a mill's water wheel hub.

Drawing 19 shows the primary circle as the mill wheel hub with paddle rotors tenoned into it on the radial alignments.


## The Daisy Wheel, Five Circles and Vesica Piscis at Ely Cathedral

The three geometrical symbols described above can all be found at Ely Cathedral. The vesica piscis can be found on all four walls of the western tower (the main entrance to the cathedral), above the western arch at the entrance to the nave and as the boundary of the exquisitely beautiful carving of Christ in Judgement in the tympanum of the Prior's door (on the cloister side of the door into the nave's south aisle), photograph 20. The daisy wheel appears in various locations throughout the cathedral including the tripartite tympanum of the Monks' door (also on the cloister side of the door into the nave's south aisle), photograph 21. The five circles are not visible as a symbol but can be recovered by analysis of the cathedral's floor plan where the configuration determines the crossing's external and internal squares and the wall alignments of the nave, choir and transepts. The drawings that follow demonstrate the function of these symbols in the design of the cathedral's floor plan.


20 THE VESICA PISCIS
THE PRIOR'S DOOR

21 THE DAISY WHEEL THE MONKS' DOOR

## Five Circle crossing and Daisy Wheel nave

Building at Ely began in 1081 under Abbott Simeon but it was sixty years before the nave was completed in 1140. The building's design is governed by two archetypal geometries, the five circle square or ad quadratum geometry and diasy wheel or ad triangulum geometry. Ad quadratum, towards quadration, is used to define the cathedral's cruciform crossing where the choir runs east, the nave west and the transepts north and south with all four arms of equal width. Ad triangulum, towards triangulation, determines the length of the nave. The crossing, which is the epicentre of the cathedral, is the first element of the design and the nave, which is longer, is a westward development from it.

Drawing 22 shows the five circle, ad quadratum, geometry superimposed upon the Romanesque cathedral floor plan. The centre line, which runs east west from choir to nave, and the perpendicular, which runs from north transept to south transept, intersect at the epicentre of the cathedral's plan. The primary circle is drawn from the intersection and is cut by the centre line and perpendicular at its north, east, south and west poles. Four identical circles are drawn from the poles and where these circles intersect, at the small red arrows, they define the exterior angles of the crossing. The four circles also cut the primary circle at eight points of intersetcion, marked by small black arrows, and alignments drawn through these intersections determine the inner faces of the choir, nave and transept walls. The five circles of the ad quadratum geometry are all drawn to a radius of 44 feet ( 88 feet diameter), making the choir, nave and transepts 88 feet wide at the external face of the walls.

Drawing 23 shows a three daisy wheel sequence constructed along the nave's east west centre line to a radius of $381 / 2$ feet ( 77 feet diameter), making the nave 77 feet wide between the internal face of the walls. The first radius is drawn from the alignment of the northern and southern transepts' western walls on the nave centre line, marked by a small red arrow.

Unlike the majority of English cathedrals, Ely and neighbouring Peterborough Cathedral are not vaulted in stone but have boarded and painted timber roofs, Peterborough's being original and Ely's being painted from the Victorian period. However, Ely's northern and southern arcades are vaulted in masonry, generating a triforium floor level above the vaults along both sides of the nave. Building the cathedral in this way allowed the nave's northern and southern walls to be constructed independently and this is probably the reason that the two walls drift apart slightly as they run from the crossing, where the nave's construction began, towards the west. This, in turn, made it necessary for adjustments to some pier locations. Nevertheless, the triple daisy wheel sequence, which is clearly influenced by the ecclesiastical concept of the Trinity, the cathedral's full title being The Cathedral Church of the Holy and Undivided Trinity, fits the arcade and bay rhythm alignments of the nave perfectly. Or, the reverse is true, that the structure stands upon a geometrical foundation that determines the positions of every element of the building.

The geometries shown in drawings 22 and 23 are both primary geometries from which secondary geometries can be constructed. The secondary constructions are shown on the following pages.


22 THE FIVE CIRCLE CROSSING DEVELOPMENT


23 THE DAISY WHEEL NAVE DEVELOPMENT

The secondary geometries refine the primary geometries by giving greater detail. In drawing 24 the secondary geometries define the terminations of the transepts and in drawing 25 the location of the cylindrical and angular piers along the arcades.

Drawing 24 shows how the secondary geometry can be developed from the five circle geometry. The primary circle is cut at its north, east, south and west poles by the centre line and perpendicular, the four points marked by small pale blue and dark blue arrows. The four outer circles intersect at the corners of a perfect square, shown in blue tone and marked at its corners with small red arrows. The centres of the square's sides are, simultaneously, the primary circle's poles. Two arcs of circle are drawn with their pins on the dark blue arrows and their pens passing through the red arrowed points on the opposite corners of the square to give the length of the north and south transepts. Two further arcs of circle are drawn from the pale blue arrows to give the length of the choir and a termination within the nave. The four arcs mark the corners of the crossing's cruciform plan.

Photographs 20 and 21 show the tympana of the Prior's Door and Monk's Door respectively, both doors entering the southern aisle of Ely's nave from opposite ends of the now lost cloisters. Both tympana, which face into the cloisters, feature geometrical constructions, the Prior's Door a vesica piscis mandorla encompassing the figure of Christ and the Monks' Door two precision daisy wheels. On the opposite side of the wall, inside the nave, the floor is laid out to daisy wheel geometry. Drawing 25 shows the triple interlaced daisy wheels that determine the nave's width, length and arcade alignments. A secondary geometry is drawn between the wheel's vertical petal tips and the nave's centre line to generate a triple diamond sequence.


24 THE FIVE CIRCLE CROSSING DEVELOPMENT

Cylindrical and angular piers alternate along the arcades in a classic parade of circularity and angularity, generating pairs of identical form across the nave's width. The cylindrical piers stand on compass geometry, four on each daisy wheel's circumference and two on each wheel's vertical diameter while the angular piers, conversely, stand on the diamond sub-geometry. The bay rhythms occur where the daisy wheel and diamond geometries intersect the arcade alignments. Any doubts regarding the validity of daisy wheel geometrical design at Ely must be weighed against the tangible evidence of precision cut examples just above eye level at either side of the tripartite Monks' Door. Further examples of daisy wheels can be found in other parts of the cathedral, for example above the door into the spiral stairs in the North Transept and on the triforium level.


25 THE DAISY WHEEL x 3 NAVE DEVELOPMENT

## Daisy Wheel six point rectangle geometry in the Barley Barn at Cressing Temple

The Barley Barn, built by the Knights Templar at Cressing Temple in Essex, is dendrochronologically dated to 1220AD. This was before the discovery of the English tying joint though the joint can be seen in the neighbouring Wheat Barn which was built in 1260AD. Because the Barley Barn lacked the strength of the tying joint the carpenters employed distinctive double tie beams with X bracing on the two midstrey trusses to ensure their stability. Photograph 26 shows the barn's interior with light from the open midstrey doors on the right. Photograph 27 shows the $X$ and $Y$ braced trusses enlarged (the $Y$ braces are inverted $\boldsymbol{K}$ and $\boldsymbol{\lambda}$ ).


26 THE BARLEY BARN INTERIOR


27 THE BARLEY BARN MIDSTREY TRUSSES

Drawing 28 shows the six point rectangle with its boundary passing through the daisy wheel's petal tips. It can be seen that the width of the rectangle, which equals the wheel's horizontal diameter, is less than the rectangle's height, which is less than the wheel's vertical diameter. Having a width greater than its height is an appropriate start for a large barn design. The drawing also shows that an equilateral triangle can be drawn from the rectangle's base line to the centre of it's top line. The equilateral is the first step in the geometrical design.

Drawing 29 shows the development of the barn's wall plate. A line is drawn between $A$ and $B$ and another between C and D, both lines linking petal tips on the daisy wheel's circumference. The lines cut the rising angles of the equilateral at two points and a horizontal drawn through these points generates the level of the wall plate (shown in a solid black line).

Drawing 30 shows how the barn's roof pitch is developed by linking the wall plate to the equilateral's apex and how the barn's full section is defined within the six point rectangle.


28 SIX POINT RECTANGLE \& EQUILATERAL


29 DEVELOPMENT OF WALL PLATE


30 DEVELOPMENT OF THE BARN'S ROOF \& FULL SECTION

Drawing 31 shows the wall plate and principal rafters that rise from it to the barn's ridge. Drawing 32 shows the addition of the double tie beams which are placed as tangents above and below the daiy wheel's horizontal vesicas and the arcade posts which plumb down from where the upper tie meets the roof plane. Drawing 33 shows the brace geometry which runs at mirror image angles drawn between the six point rectangle's corners and the daisy wheel's petal tips. The geometry generates two pairs of parallel lines, see $A B$ and CD which are in mirror image to EF and GH. The lines intersect between the double tie beams to generate the $X$ and inverted $Y$ braces. An $X$ brace is shown on the left side of the truss and a $Y$ brace on the right. In reality one midstrey truss has two $X$ braces and the other two $Y$ braces. The collar is drawn where lines CD and EF cut the roof pitch.


31 WALL PLATE \& PRINCIPALS


32 PARALLEL TIE BEAMS


33 BRACE GEOMETRY


34 GEOMETRY SUPERIMPOSED ON MEASURED DRAWING
Drawing 34 shows the geometrical frame superimposed over Essex County Council's measured drawing. It can be seen that the barn's full width and height fits the width and height of the daisy wheel's six point rectangle exactly (the geometry rises from the upper face of the sills). This drawing focusses on the bracing between the upper and lower tie beams. The brace alignments run in diagonal pairs between the corners of the six point rectangle and the tips of the daisy wheel petals along the base and head line of the rectangle and intersect between the double tie beams. The alignments are shown in solid green and blue line and it can be seen in the drawing that they pass exactly along the outer edges of the $X$ braces and meet the horizontal parallels of the double tie beams which, in turn, fit exactly to the daisy wheel's horizontal vesicas. At the time of writing in 2017, this tightly braced core sector of the frame remains identical to the geometry after almost 800 years, a tribute to Knights Templar design and carpentry skills.

## Five Circle geometry at Tŷ Mawr, Castle Caereinion, Montgomeryshire

Tŷ Mawr (the Great House) is a timber framed aisled hall house, dendrochronologically dated to 1460 , of box framed construction except for a single cruck arch spanning the hall. At some unknown time in its history the cruck arch failed in both aisles at the point where the arcade plate braces met the cruck and, as a result of this the hall's outer aisle walls were demolished and rebuilt closer to the nave. The upper, central sector of the cruck survived between the arcade plates. Over the years this narrower building continued to slip down the social scale until it was discovered clad in corrugated iron and in use as a barn by Peter Smith (of the Royal Commision on the Ancient and Historical Monuments of Wales). The frame was recorded and during this process a geometrical symbol, discovered on the inner face of the north gable's eastern aisle post, was recorded as a video image, photograph 35. Drawing 36 shows the symbol's geometry.


Drawing 37 shows the construction of the five circles. In the first stage, three circles are drawn along a horizontal centre line and, where they intersect above and below the line, pairs of arcs are drawn (upwards and downwards in green line). A vertical line drawn through the intersections of the arcs generates a perpendicular to the centre line. All five circles and the symbol's short arcs are drawn to the same radius, the dividers indicating the radius on the left and location for drawing an arc on the right, at the green arrow.


37 FIVE CIRCLE GEOMETRY


38 TŶ MAWR SYMBOL'S FUNCTION AS A DESIGN ICON
Drawing 38 demonstrates how Tŷ Mawr's symbol functions as a design icon, a geometrical shorthand for the design methodology of the complete house. The geometry of the symbol can be developed first by drawing vertical bisections through the vertical vesicas on the left and right. The bisections cut the horizontal centre line at the centre of each vesica, at geometrical locations that are crucial to determining the pitch of the roof. The pitch is drawn from the top centre of the symbol to pass through the crucial intersections and downwards until they meet a horizontal tangent to the symbol's base. A secondary circle (in dashed line), drawn from the symbol's base, determines the building's ground level. Vertical tangents to this circle and the symbol generate the positions of the frame's massive spere posts. At the level of the roof triangulation's base, marked by arrows, the spere post sections change from their lower octagonal sections to cruciform upper sections.

## Parallel Worlds

Throughout the medieval period geometry was the state of the art design system and it is clear from the above examples that geometrical symbols played a vital role in the design of buildings that varied in scale from hall houses to monastic barns and cathedrals. The crystal clear logic of geometrical thought and its precision practical application in building design were the absolute antithesis of unquantifiable superstition and magic. This may be why geometrical symbols were used as defensive totems in the battle between enlightenment and ignorance.

NOTE
The actual geometrical symbol recorded on the inner face of the eastern aisle post in the north gable of Ty Mawr was lost and presumed destroyed during a clean up at the end of the building's repair project when amounts of waste wood were burned. Thankfully, the symbol was recorded as a measured drawing and on a video clip (the image shown on page 16) and these allow the reconstruction of the geometry. A fake reproduction carving was made in the same location on the replacement timber but the loss of the original symbol is catastrophic for several reasons. The primary reason is that Ty Mawr was the first house in Wales to have a geometrical symbol that allowed the full development of a floor plan, section and long elevation. It was, therefore, the perfect historic building in that its frame, dendrochronologically dated to 1460, was embodied in the shorthand geometrical clue of it's symbol and through analysis of the symbol the mindset of the carpenter who framed it could be understood. The second reason why the symbol's loss is catastrophic is that there was clearly no serious strategy for its protection. And the third reason is that without a strategy for protection, fragile historic elements are vulnerable through ignorance of their value. The fact that the meaning of something is unkown should be a warning sign: first for protection and then for research.

